RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021

FIRST YEAR [BATCH 2020-23]

Date :14/08/2021 **MATHEMATICS(General)**

Time:11 am - 1 pm **Paper** : II Full Marks: 50

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group - A (Ordinary Differential Equation)

Answer any **three** questions from Q.1 to Q.5.

[3x5=15]

1. Find the orthogonal trajectories to the family of circles that pass through the origin and have their centres on x-axis. [5]
2. Solve: $p^2(2-3y)^2 = 4(1-y)$, where $p = \dfrac{dy}{dx}$. [5]
3. Show that $(4x+3y+1)dx + (3x+2y+1)dy = 0$ represents a family of hyperbola. [5]
4. Solve: $6\dfrac{d^2y}{dx^2} + 17\dfrac{dy}{dx} - 14y = \sin 3x$. [5]
5. Solve: $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2\sin(\ln x)$. [5]

Group - B (Calculus)

Answer any **seven** questions from Q.6 to Q.15.

[7x5=35]

6. (a) Define bounded sequence and give an example of bounded monotonic increasing sequence. [2]

   (b) Prove that every convergent sequence of real numbers is bounded. [3]

7. Let $\{X_n\}$ and $\{Y_n\}$ be sequences of real numbers such that $\lim\limits_{n\to\infty} X_n = X$ and $\lim\limits_{n\to\infty} Y_n = Y$, then show that $\lim\limits_{n\to\infty} X_nY_n = XY$. [5]

8. State the *Cauchy's* principle of convergence for series. Use it prove that the series $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ is divergent. [1+4]

9. If $f : I \longrightarrow \mathbb{R}$ has derivative at $c \in I$, then show that f is continuous at c. Is the converse true, justify your answer. [3+2]

10. State *Lagrange's* Mean value theorem. Use it to prove that $-x \leqslant \sin x \leqslant x$ for all $x \geqslant 0$. [1+4]

11. Prove that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$, is continuous at $(0,0)$. [5]

12. If $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$, then show that $\frac{\partial^2 f}{\partial x \partial y}(0,0) \neq \frac{\partial^2 f}{\partial y \partial x}(0,0)$. [5]

13. If $u = \tan^{-1} \frac{x^3+y^3}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. [5]

14. Evaluate $\iint x^2 \, dx \, dy$ over the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$. [5]

15. Find the oblique asymptotes of the curve $y = \frac{3x}{2} \log\left(e - \frac{1}{3x}\right)$. [5]